Group Members:

1. Heat Engine

A diesel engine performs $2200.J$ of mechanical work W and discards $4300.J$ of heat Q_C through each cycle. (Please play attention to sign convention to all heats and work.)

a. Calculate the amount of heat Q_H must be supplied to the engine in each cycle from burning of its fuel?

heat discarded or released is "−", so $Q_c = -4300J$

Work done or performed is "+", so $W = +2200J$

Then, by conservation of energy, we have

$W = Q_c + Q_H$

$+2200J = -4300J + Q_H$

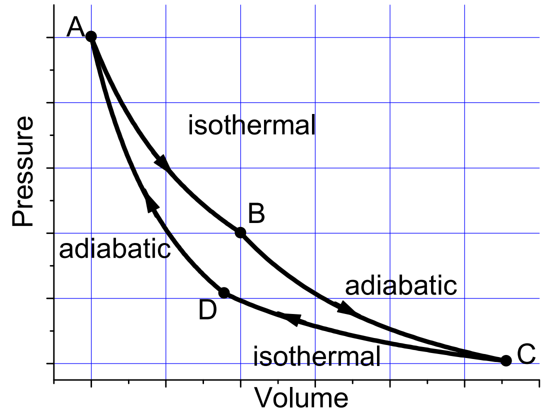
$Q_H = 2200J + 4300J = 6500.J$

b. What is the thermal efficiency e of the engine?

$$e = \frac{W}{Q_H} = \frac{2200J}{6500J} = 33.85\%$$

2.

A Carnot cycle operated between two temperature reservoirs at T_H=500ºC and T_L=200ºC has one mole of an ideal monoatomic gas as its working substance. You are given the following parameters for the cycle: V_A=10.0L, V_B=20.0L. Calculate a) the net work done by the gas per cycle, b) the heat absorbed by the gas per cycle, c) the heat expelled by the gas per cycle. d) What is the efficiency of this cycle?
[$1L = 10^{-3} m^3$, 1 atm=$1.013\times10^5\,Pa$]



To get the volume at state C and D, we can use the adiabatic processes:

$$T_C V_C^{\gamma-1} = T_B V_B^{\gamma-1}$$

$$V_C = V_B\left(\frac{T_H}{T_L}\right)^{1/(\gamma-1)} = 20L\left(\frac{500+273.15}{200+273.15}\right)^{\frac{3}{2}} = 41.78L$$

Similarly,

$$T_D V_D^{\gamma-1} = T_A V_A^{\gamma-1}$$

$$V_D = V_A\left(\frac{T_H}{T_L}\right)^{1/(\gamma-1)} = 10L\left(\frac{500+273.15}{200+273.15}\right)^{\frac{3}{2}} = 20.89L$$

Note that since we really only need the ratio V_D/V_C, we can simply divide the above two equations (without doing the numerical calculations) to get

$$\frac{V_D}{V_C} = \frac{V_A}{V_B} = 0.5.$$

a) Net work done by the cycle:

Isothermal branches (A→B, C→D) ($\Delta U = 0$):

$$W_{AB} = nRT_H \ln\left(\frac{V_B}{V_A}\right) \qquad \text{(work done by)}$$

$$= 1mol(8.314J/mol\cdot K)(773.15K)\ln(20/10) = +4.456kJ$$

$$W_{CD} = nRT_L \ln\left(\frac{V_D}{V_C}\right) \qquad \text{(work done on)}$$

$$= 1mol(8.314J/mol\cdot K)(473.15K)\ln(0.5) = -2.727kJ$$

Adiabatic branches (B→C, D→A) ($Q = 0$):

$$W_{BC} = -\Delta U_{BC} = -nC_V(T_C - T_B) = -(1mol)\frac{3}{2}(8.314J/mol\cdot K)(473.15-773.15)K = +3.741kJ$$

$$W_{DA} = -\Delta U_{DA} = -nC_V(T_A - T_D) = -(1mol)\frac{3}{2}(8.314J/mol\cdot K)(773.15-473.15)K = -3.741kJ$$

Thus,

$W_{net} = 4.456kJ - 2.727kJ + 3.741kJ - 3.741kJ = +1.729kJ$ (net work is done BY gas).

b) Now we calculate the heat transfers:

A→B: isothermal. $Q_{AB} = W_{AB} = +4.456kJ$
B→C: adiabatic. $Q_{BC} = 0$
C→D: isothermal. $Q_{CD} = W_{CD} = -2.727kJ$
D→A: adiabatic. $Q_{DA} = 0$
Thus,

$$Q_{abosorbed} = Q_{AB} = +4.456kJ$$

And the system releases heat during only the process C→D so that

$$Q_{released} = Q_{CD} = -2.727kJ$$

d) The efficiency of this heat engine is

$$e = \frac{|W_{net}|}{|Q_{absorbed}|} = \frac{1.729kJ}{4.456kJ} = 0.3880 = 38.8\%$$

3. Entropy

The volume of one mole of a monoatomic ideal gas is being expanding from V to $5V$.

a. Calculate the entropy change if the process is an isothermal expansion.

$n = 1$ mol, $C_V = \frac{3}{2}R$

$$\Delta S = nC_V \ln(T_f/T_i) + nR\ln(V_f/V_i)$$

For isothermal process, $T_f = T_i = T$

So,

$$\Delta S = 0\ J/K + (1\ mol)(8.314\ J/K\cdot mol)\ln(5)$$

$$= 13.4\ J/K$$

b. Calculate the change in entropy if the process is a reversible adiabatic expansion.

$dS = \frac{dQ}{T}$ Since $dQ = 0$ $\Delta S = 0\ J/K$

We can also use the above equation to explicitly calculate it,

$$\Delta S = nC_V \ln(T_f/T_i) + nR\ln(V_f/V_i)$$

For adiabatic processes,

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$\frac{T_f}{T_i} = \left(\frac{V_i}{V_f}\right)^{\gamma-1}$$

$$\Delta S = nC_V(\gamma-1)\ln\left(\frac{V_i}{V_f}\right) + nR\ln(V_f/V_i)$$

$$\gamma - 1 = \frac{C_p}{C_v} - 1 = \frac{R + C_v}{C_v} - 1$$

$$= \frac{R}{C_v} \cancel{+1} \cancel{-1}$$

$$= \frac{R}{C_v}$$

So, $\Delta S = n \cancel{C_v} \left(\frac{R}{\cancel{C_v}}\right) \ln\left(\frac{V_i}{V_f}\right) + nR \ln\left(\frac{V_f}{V_i}\right)$

Since $\ln\left(\frac{V_i}{V_f}\right) = -\ln\left(\frac{V_f}{V_i}\right)$

$\Delta S = 0\,J$